Chapter 3: Chain Reaction and Point Kinetics

All right, so, if you recall the first two lectures, they were quite heavy on the mathematics. Let's take a little break from that, shall we? This lecture aims at showing you the why of this course. We'll define a chain reaction, see the different ways to get one, and talk about point kinetics. Basically, it represents the (very) simplified theory allowing us to answer the important question: "how to control your nuclear reactor?"

3.1. Chain reaction

Chain reactions. You have all heard the word, no doubt about that. It is after all an everyday concept. For example, fire is a chain reaction. Heat causes a chemical reaction, called combustion), that in turn produces heat, which causes combustion, and so forth. So, when physicists discovered that neutron-induced fission also emitted some neutrons, they of course realized that you could get a chain reaction. The energies released by these mechanisms being so much greater than the chemical energies (fire for example). Perfect for awesome peaceful source of energy, or for phenomenal weapon…
This course focuses on how to know your neutron flux in a reactor in order to control it. Hence, the rate of reaction must be controlled. For a weapon, fast amplification of the reaction is required.
The behavior of the reaction will depend on one factor, k.

Let us consider ω as the probability that a neutron induces a fission reaction. The opposite probability, $(1-\omega)$, is the probability that this neutron is absorbed by the system or escapes (i.e. is captured by the outside). I have already mentioned that a fission reaction emits on average ν new neutrons.
Consequently, the product $\nu.\omega$ represents the average number of neutrons being created as a result of a neutron placed in the system. Since the neutrons (ν) resulting from this fission reaction can also induce a fission reaction with a probability ω, we can also interpret k as being the average number of fission reactions that result from an initial fission reaction.

So, if we have a lot (N) of fission reactions, we can see that:
Step 1: N fissions
Step 2: $N * k$ fissions
Step 3: $N * k^2$ fissions
Step 4: $N * k^3$ fissions
.
.
.
Step i: $N * k^{i-1}$ fissions

We can therefore see that the behavior of the chain reaction will depend on the value of k. Indeed:

- If $k > 1$, the reaction will accelerate
- If $k = 1$, the reaction will be self-sustained and the rate will be constant
- If $k < 1$, the reaction will die away, eventually.

The first case ($k > 1$) is what is needed for weapons. The second one is known as the *critical configuration*, and is the configuration that exists in a reactor in stable operation (in a perfect world, since being exactly on $k = 1$ is not achievable). If you want to start p a reactor, or to increase its power, you have to temporarily place it in a slightly *supercritical state* (k a little over 1). If on the other hand you want to shut it down or decrease its power, well obviously it is placed in a *subcritical state* ($k < 1$).
The most common method to adjust this factor k is to use control rods, containing absorbing materials (prone to absorb neutrons, thus decreasing the factor k). You can change their position (extract or insert them) which impacts the global neutron population in the reactor core, thus modifying the factor k.

The factor ν is in the region of 2.4 for uranium 235, so achieving criticality ($k = 1$) requires a probability ω of $\frac{1}{2.4} \cong 42\%$, and if we wanted to create a weapon, this probability would be a little higher. The question was, in the late 1930s, if that was possible. The information those physicists had at the time was:

- Uranium has three natural isotopes: uranium 238, uranium 325, uranium 234. Uranium 238 cannot undergo fission except with very energetic neutrons. Uranium 235 can fission with any incident neutron. Uranium 234 exists only in negligible quantities.
- Unfortunately, the fissile 235 isotope made up for only 0.72% of all natural uranium (1 nucleus of 235 for every 139 nuclei of 238).
- Neutrons emitted by fission are emitted with an energy of around $2\ MeV$, which represents around $20000\ km.s^{-1}$
- At this energy, the cross-sections of both uranium 235 and 238 are of the same order of magnitude.
- In a low capture material, it is possible to decelerate neutrons by successive scattering until they nearly reach thermal equilibrium with matter. Thermal neutrons have an energy around $\frac{1}{40}eV$ (that is around $2\ km.s^{-1}$)
- For neutrons in this energy range, the cross-section of uranium 235 is much larger than that of uranium 238 (approximately 250 times larger)

So, that leaves us with two options to create a chain reaction:

- Enriched uranium and fast neutrons.
  - This uses neutrons at the energy at which they are produced by fission reactions, without decelerating them, and using a fuel that is highly enriched in fissile isotope (uranium 235 or plutonium 239)
- Unenriched uranium and thermal neutrons.
  - This uses neutrons that have been slowed and thermalized by a material called moderator. A fuel poor in fissile isotopes can be used. Hell, even natural uranium can be, if your moderator is efficient enough. Little “digression” here: The CANDU kind of reactors (Canada) use heavy water as a moderator/coolant (in lieu of light water in PWRs and BWRs) as to avoid enriching their uranium.

Enriching uranium is not that easy and quite expensive. On the other hand, producing heavy water is also expensive.
    - Even in the case of natural uranium, a slow neutron passing through uranium has a higher probability of being absorbed by the 235 and causing fission than being absorbed by the 238 and not causing fission (139 times more 238 nuclei, but 250 times more likely to interact with 235)

The second option is very widely used, with the difference that “unenriched” becomes “slightly enriched” very often. By slightly enriched, understand 3 to 4% of uranium 235.

Both options were introduced during the Second World War. The natural uranium and thermal neutrons was used in Fermi’s pile. This is, with some modifications, the method currently used for a nuclear reactor. This method cannot be used directly to make a nuclear bomb: indeed, it takes too long to decelerate the neutrons enough for them to be “thermal”, and thus an efficient explosion cannot take place. Moreover, including the moderator would make the bomb gigantic.

A moderator consists of a material with low neutron capture. This avoids wasting the neutrons supplied by fission by absorbing them. This material also must be good at slowing down neutrons, i.e. scattering (the neutrons bounce around and lose a little energy every time. The more it bounces, the slower it gets). For that to happen, it means that the material is composed of light nuclei and is dense (a lot of small target).
This basically eliminates gas and favors liquid and solid.
The choice of materials for a moderator thus becomes quite restricted. The main options are::

- Liquid or solid hydrogenated materials : water for example
- Heavy water (hydrogen H replaced by deuterium D)
- Carbon (graphite)

As seen earlier with the CANDU reactors specificity, natural uranium cannot be used with light water. This is because hydrogen is captures some neutrons. Deuterium is much better in this regard (less capture, more scattering).
If you want to use light water, you must enrich your uranium (increase the percentage of uranium 235). The main advantage of light water, as I’ve mentioned earlier, is that it is cheap.

In a power reactor, a fluid must be made to circulate in order to extract the heat produced by the fission reactions. That is the coolant. It can either be gas (CO_2, He,…) or a liquid (water, heavy water, liquid metal, …)
This is basically how you design/separate different type of nuclear reactors:

- Moderator
    - Presence or absence (for fast neutron reactors)
    - Liquid, solid?
    - Which one?
- Coolant
    - Liquid, gas?
    - Which one?
- Fuel
    - Which one?
    - Which physical/chemical form?
    - Where?
- Cladding
    - Which material?

Depending on the interest, another course might be created to discuss each type of nuclear reactors in details.

3.2. Point kinetics

Considering only the factor k, as it was introduced earlier, might get you into troubles if you are trying to keep a nuclear reactor functioning properly.
In order to discuss this point, I might repeat stuff you have already seen earlier in this course, but I feel it is important to look at the whole reasoning. And repetition surely will not hurt your understanding. And repetition surely will not hurt your understanding.

3.2.1. Neutron lifetime

First, let's talk about the neutron lifetime. In a PWR reactor (Pressurized Water Reactor), it is around 10^{-5} seconds ($2.5 * 10^{-5}$ seconds to be precise). This means that after this time on average, the neutron will have disappeared (absorbed, absorbed to induce fission, or leaked out of the reactor).

3.2.2. Multiplication factor

As seen above:
$k < 1$: Subcritical, the chain reaction dies
$k = 1$: Critical, the chain reaction is nice, the power is constant
$k > 1$: Supercritical, the neutron population increases with each generation, the power increases. No good, if not controlled.

We can write the following evolution of the neutron population in the reactor with time:

$$n(t) = N * e^{k-1} * e^{\frac{t}{X}}$$

In this equation, X is the mean lifetime of the neutrons inside the reactor and N represents the original neutron population.

Question n°1 : ✪

Please demonstrate why $n(t) = N * e^{k-1} * e^{\frac{t}{X}}$

So, let's consider that $k = 1.001$. You will agree that it is difficult to get really closer to 1, in a realistic world. Let's take the PWR reactor case for example. We can write, with $k = 1.001$ and $X = 10^{-5}$ s:

$$n(t) = N * e^{1.001-1} * e^{\frac{t}{10^{-5}}}$$

Which can be written:

$$n(t) = N * e^{\frac{10^{-3}}{10^{-5}} * t}$$

And finally:

$$n(t) = N * e^{100t}$$

This means that the neutron population is *not at all* under control! After only 1 second, we have the original population of neutron (N) multiplied by e^{100} (and that is a big, big number). So the power in the reactor would increase very quickly, even though the multiplication factor is as close as possible (reality-wise) to 1.
So, what are we missing? What we'll call the *delayed neutrons*.

3.2.2. Delayed neutrons

What? A new kind of neutrons? Where do they come from ? To answer that, we must recall where we already know the neutrons can come from in a nuclear reactor. Several possibilities:

1. Fission induced neutrons: mainly, in a PWR, uranium 235 gets hits by a low energy neutron, and produce on average around $\nu = 2.4$ neutrons
2. External sources: let's forget that, we don't care here
3. Different decay reactions: instead of undergoing fission, an atom, uranium 235 for example, would absorb one neutron and release two neutrons, for example

So, what are we missing? Well, the fission of an atom creates two smaller atoms. Those (called precursors) are the ones that will release neutrons *later*, by decaying. Hence "delayed" neutrons. Indeed, the mean lifetime of a precursor is 13 seconds roughly (compared to the 10^{-5} seconds neutron lifetime in a PWR).

Delayed neutrons represents around $680\ pcm$ $(0.68\ \%)$ of the whole neutrons "produced" during a generation. This very small difference is indeed what allows us to control the chain reaction in a nuclear reactor.

Let us consider a multiplication factor k. This means that if the first generation of neutron has a population of n neutrons, then the second generation will have a population of $k.n$ and I explained previously that this made the reactor impossible to control because of the sensitivity of the factor k.

When we take the delayed neutrons into account, we can say that the first generation has n neutrons. Then, we will have $k.n.(1-\beta)$ prompt neutrons and $k.n.\beta$ new precursors, fission products that are going to undergo radioactive decay releasing a neutron. Around 13 seconds later on average, this will give way to $k.n.\beta$ delayed neutrons. It depends on the precursors but that is approximately what you will get in a PWR and is just to give you an idea, comparing 13 seconds to 10^{-5} seconds. In the previous equation, β is the following ratio:

$$\beta = \frac{Precursor\ atoms}{delayed\ +\ prompt\ neutrons}$$

Now, we can define the *reactivity* ρ as the following:

$$\rho = \frac{k-1}{k}$$

It is usually expressed in pcm, from the French language "per cent mille" (per 100,000).

I promised that this class would not have too much mathematics in it, so I'll just sum up the results. If you compute the evolution of the neutron population and of the "precursors" concentration with time, you will get a coupled differential system. This leads to what is known as the *Inhour equation* (you can also find it called *"Nordheirm equation"*).

From there, you can infer that ρ must be lower than the value of β (for ^{235}U, it is $\beta = 680\ pcm$, which is roughly equals to $1\$$. Yes, a dollar is also a unit of reactivity!). If ρ is greater than β, the delayed neutrons do not play a significant role anymore and the reactor is out of control.

This factor β of delayed neutrons seen above is an essential parameter to characterize a system in kinetic terms and, more precisely, in terms of the *risk of a criticality accident* (this is to say, an uncontrolled insertion of a high positive reactivity). This means of course that this parameter must be evaluated with the greatest care in order then to operate your reactor.
As a consequence, one has to "weight", i.e. correct for the spatial and energetic components of the neutron flux at any time and some other secondary stuff.

So, this problem applies particularly to β when you want to compute the kinetics of your reactor, it's the parameter that is most sensitive to these spatio-energetic aspects. Careful weighting must be applied according to the fission rate of each nuclide (because the individual β values are very different from a nucleus to another). On the other hand, it is also important to consider the fact that delayed neutrons are emitted at a lower energy than prompt neutrons ($400\ keV$ on average instead of $2\ MeV$).
In the case of a thermal neutrons reactor, the delayed neutrons are at less risk of escaping (absorption, leakage) in the fast region before being slowed down and more likely to provoke a fission. They are therefore more "efficient" than prompt neutrons. They will not, however, cause rapid fissions, since they are emitted at a lower energy. All this is what leads to the use of an "effective β" in calculations. This β_{eff} is slightly different from the "mean β" obtained by weighting the "β_k" of the various $(1, 2, \ldots, k)$ fissile nuclides by the fission rates of each one. I am barely mentioning that for general understanding, don't worry if it's not crystal clear.

Well, this ends the third lecture. If you have any question, please let me know directly or post a thread in the dedicated subreddit. Do not forget, and I can't stress this enough: if you have a question, then someone else in the class is wondering the same thing, or should be. Therefore, asking it will help you and others.

Point kinetics. I have barely scratched the surface of this subject. If you really wish to know everything about point kinetics, I may come back to it at the end of this course. However, what is there is more than enough to understand what's going on in the reactor. It is not very important for the following of the course (we actually probably won't mention it again).

There is another thing that I should repeat. If you do not understand something, do not feel like it's your fault, and do not give up. It merely means that my explanations were not good enough. I will gladly upgrade the class by taking into account your suggestions and remarks.