UNIVERSITY OF REDDIT **Nuclear Reactor Physics : Neutronics**

Chapter 6: Neutron Slowing Down – Part I

4.1. Introduction

In thermal reactors, in order to achieve criticality, we want to slow the fission neutrons down to thermal energies. As seen in a previous lecture, this means making the neutrons go from around 2 MeV to 0.025 eV. We use the pool principle: balls which collide lose some of their speed. And, as I've briefly mentioned before, the absorption resonances must be avoided (see lecture 8).

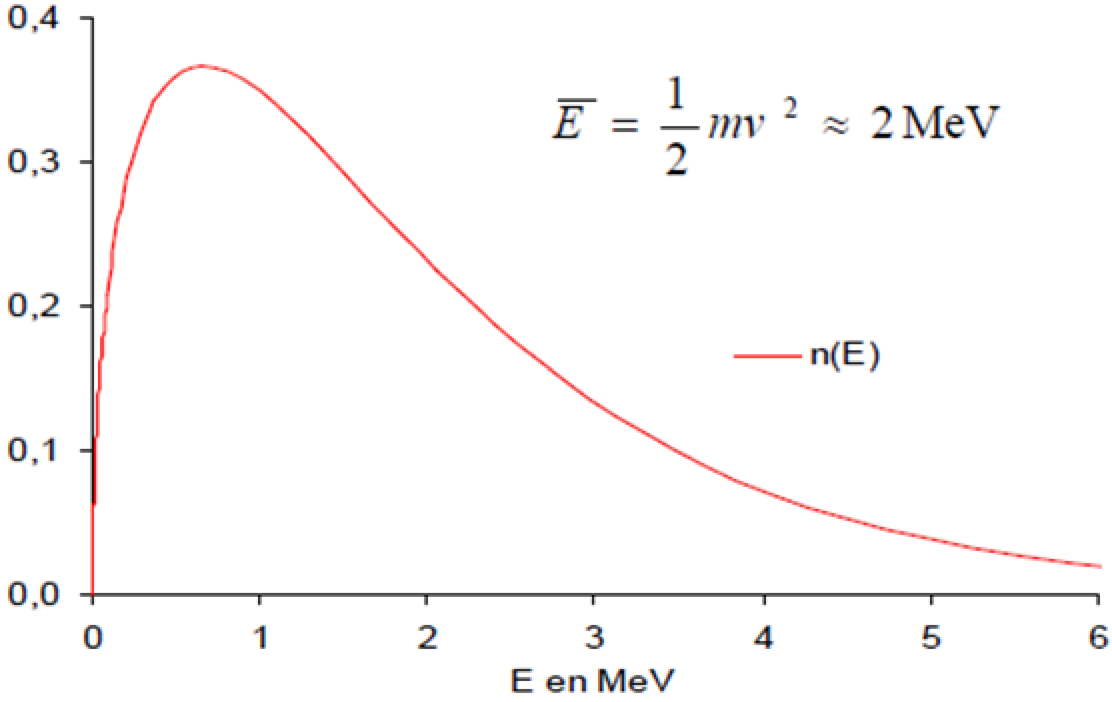


Fig 1. The fission spectrum

We can recall this graph (fig. 1), which represents the fission spectrum. This is to say, it shows the speeds (consequently energies) at which the neutrons are released during a fission reaction. On average, it's around 2 MeV.

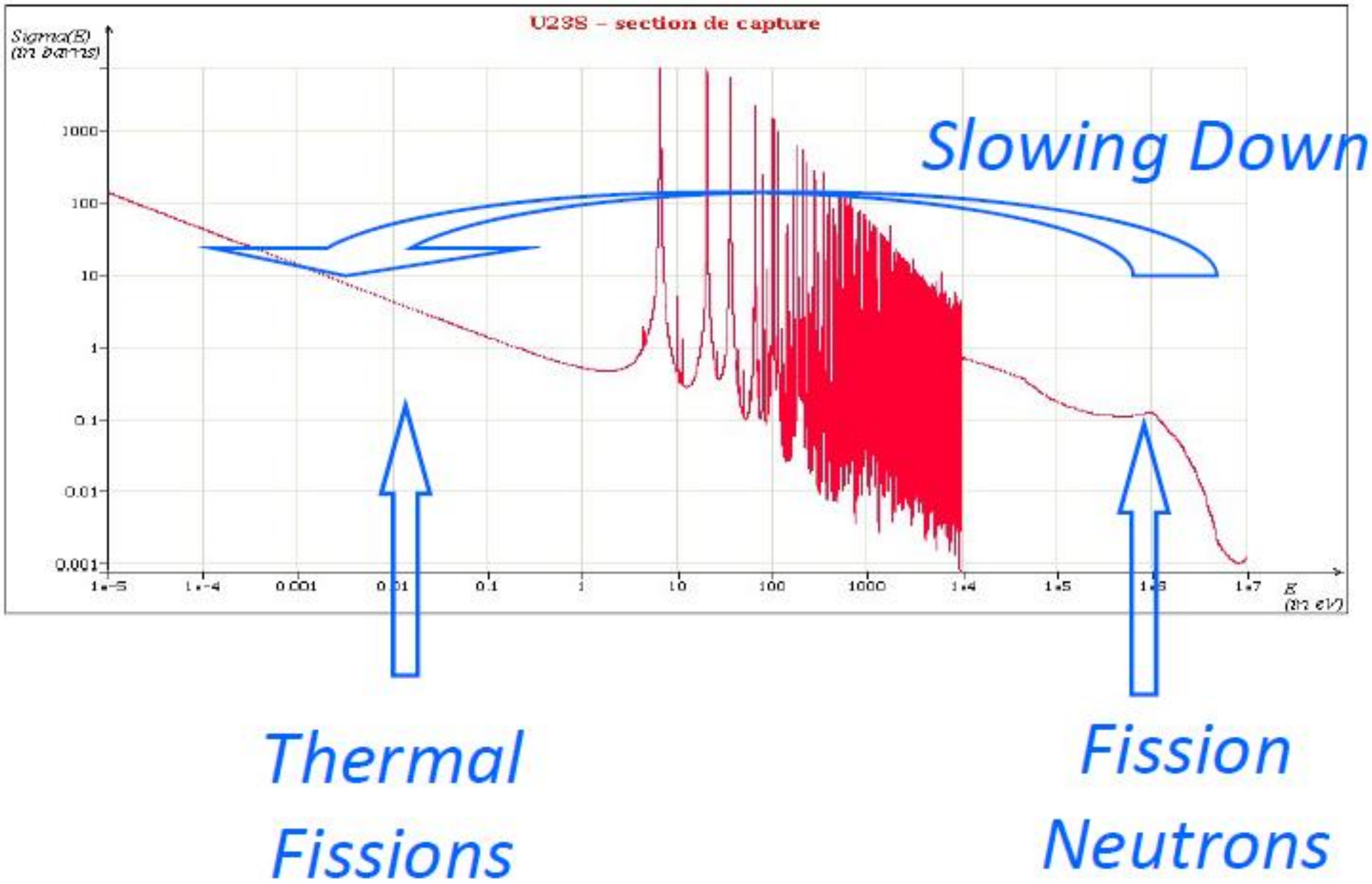
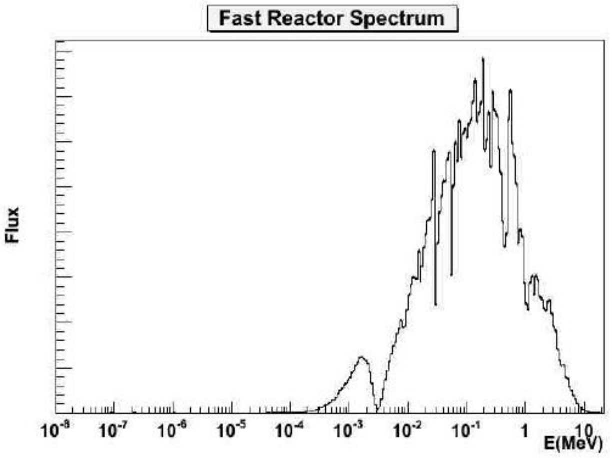
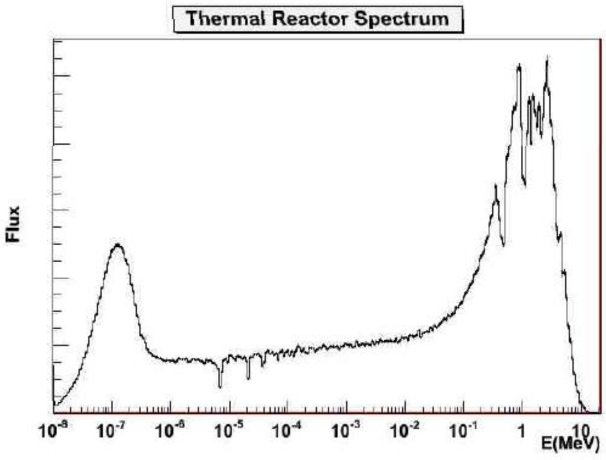


Fig 2. Capture cross-sections of U238 vs energy of the neutrons

This second graph (fig. 2) displays the capture cross-sections of U238 depending on the nergy of the neutrons. We can see that fast neutrons (fission neutrons) have a relatively small chance of being absorbed by U238. Indeed, above 1 MeV, the cross-section decreases. This is why fast reactors with Uranium are a good option for the future (main disadvantage being the proliferations concerns), but that is another story that I plan to talk about in a later course on nuclear reactors designs. What else do we see in this graph? Well, we have, in the slowing down regions, the cross-sections going crazy, up and down. Those are of course the resonances. We see that for some energies, the peaks are very high, thus the probability of the neutron being absorbed is high, if by chance its energy hit one of those resonances.

We want to compute the spectrum (in other words the energy distribution) of the flux, particularly in the so-called slowing-down region (for E > 1 eV)





4.2. Principal reactions

In the slowing down region, we have a competition between diffusions, in the moderator, and absorptions, in the fuel. The scattering interaction can be potential or resonant. What do I mean by that?

Well, first, we have to refresh our memories, and talk about elastic and inelastic scattering.

A collision is elastic when kinetic energy is conserved, inelastic otherwise. So, if some of the energy of the incident particle has gone towards modifying the internal state of the target. To come back to the pool principle, the collision between two ivory balls would be (nearly) elastic, but if those balls where made of modeling clay for example, the collision would then be inelastic. In particle physics, scattering is inelastic if the target nucleus, which is initially at its fundamental energy level, reaches an excited state after interaction with the neutron. Being excited, this nucleus will later decay by gamma emission.

The potential scattering is always elastic. It corresponds to a single diffusion of the wave associated with the neutron by the potential field of the nucleus. Its cross-section is of the order of a few barns.
The resonant scattering corresponds to the absorption of the incident neutron, the formation of a compound nucleus and then the re-emission of a neutron. If, after ejection of the neutron, the target nucleus is at the fundamental level (back to initial state), the scattering is elastic. Else, it is inelastic.

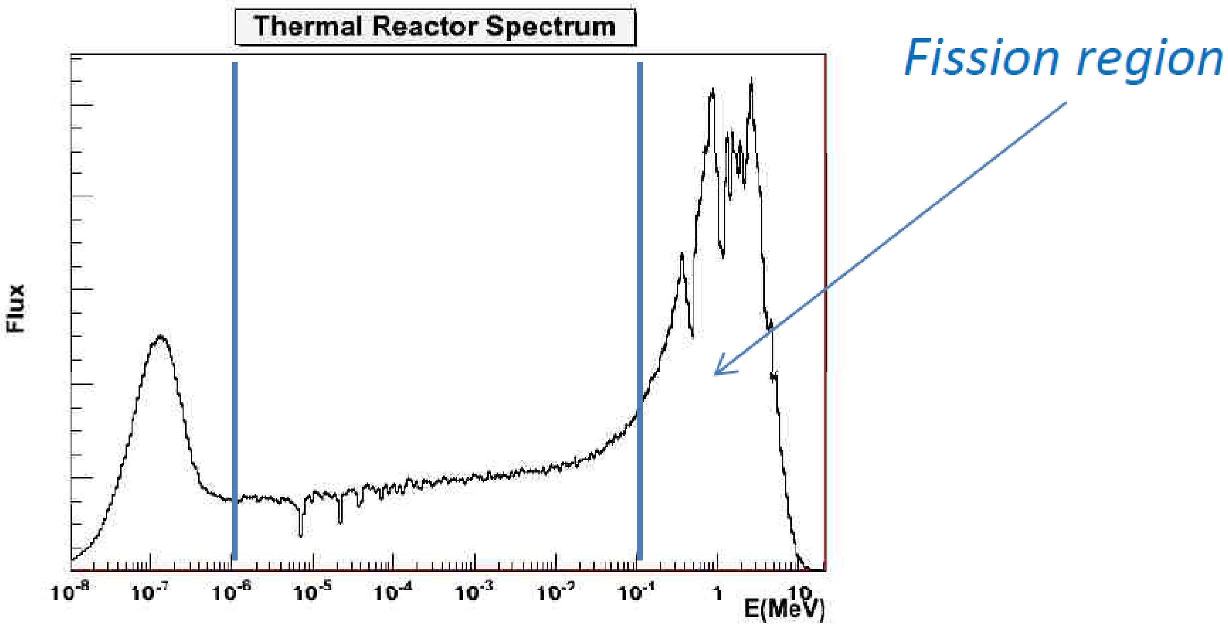
So, in potential scattering, the incident neutron leaves, while in resonant scattering, any neutron of the compound nucleus is ejected.

Elastic scattering has no threshold. It can thus occur with neutrons of any energy. Inelastic scattering, on the other hand, has a reaction threshold: the incident neutron must contribute at least the energy required to take the target nucleus from the fundamental state to the first excited level. This threshold is a few MeV for light nuclei and a few tens of keV for heavy nuclei. So, in reactors, inelastic scattering will mainly be observed in the fuel materials (U238 particularly)

Elastic scattering plays the most important role in neutrons slowing down, especially in thermal neutron reactors containing a moderator.

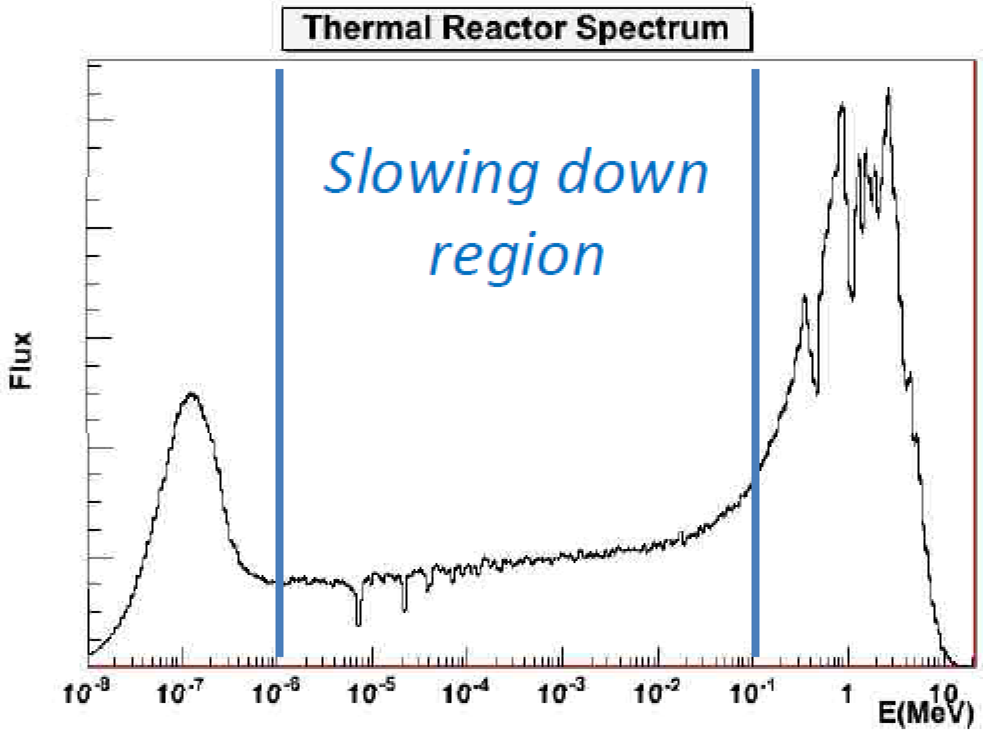
We can identify three regions in a thermal reactor spectrum.

- The fission region : E > 100 keV



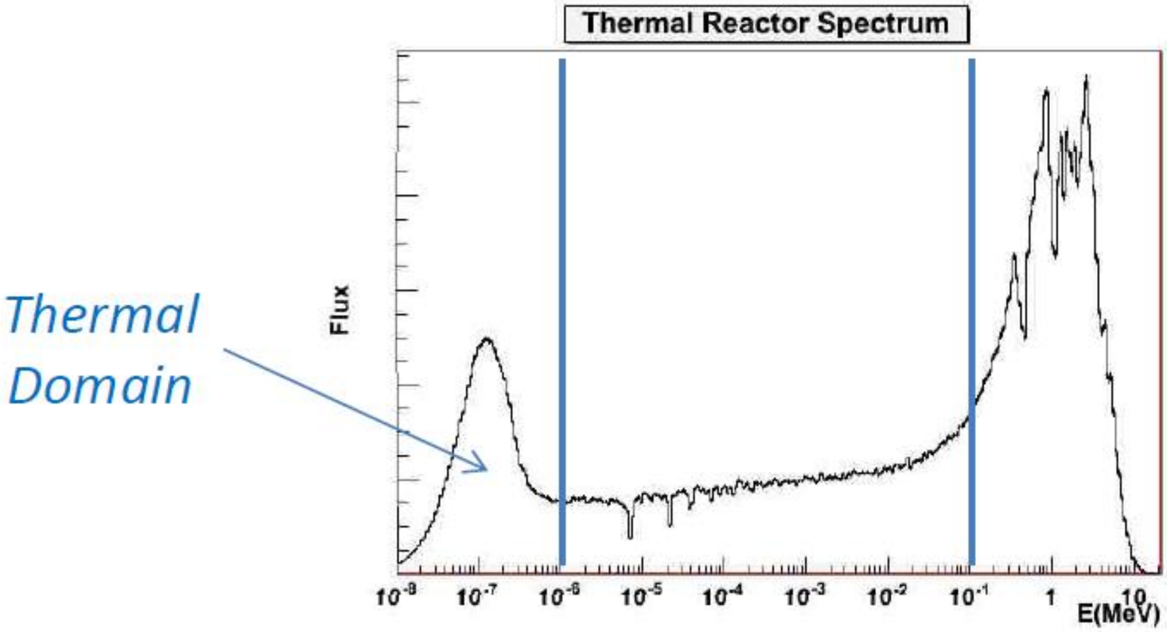
  - The flux almost follows the fission spectrum (see fig. 1), but for the “holes” due to the elastic scattering of the oxygen
  - Reactions occurring: inelastic scattering, anisotropic elastic scattering
  - Unresolved resonances

- The slowing down region : 1eV < E < 100 keV



- ➢ The flux is roughly proportional to $\frac{1}{E}$ (but for the “holes” due to the resonant absorption)
- ➢ Reactions: isotropic elastic scattering, resonant absorption.
  - ▪ The neutrons can only lose energy.
- ➢ Resolved resonances

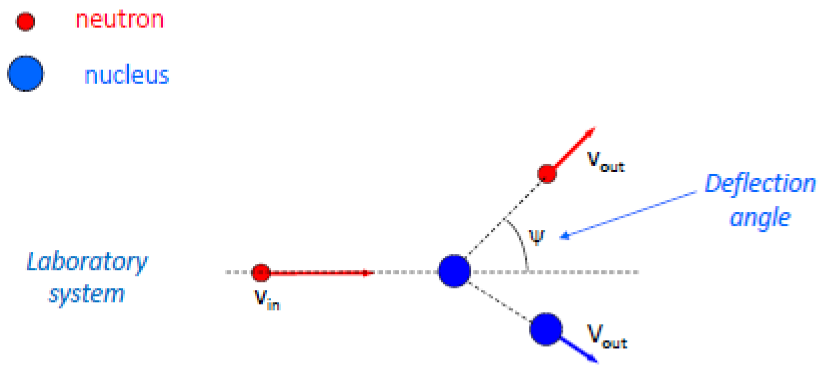
- ❖ The thermal region: E < 1 eV



- The flux approaches a Maxwell distribution in thermal equilibrium with the medium
- Neutron scattering must take into account the thermal motion of the medium, molecular bindings, vibrational modes, etc.
    - The neutron can lose or gain kinetic energy

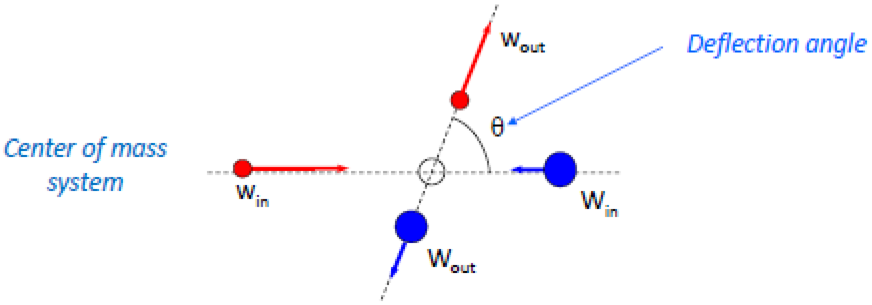
4.3. Elastic scattering

We consider the slowing down region, and we assume the nuclei to be at rest. In the laboratory system, we have the picture:



We have a rotational symmetry with relation to the axis, and conservation of total kinetic energy and momentum holds.

In the center of mass system, the total momentum is zero: the neutron (and nucleus) velocity does not change its speed, only its direction. The result of the collision is a simple deflection θ.



With $A = \frac{M_{nucleus}}{M_{neutron}}$, we can write than:

$$\vec{v}_{CM} = \frac{1}{A+1}\,\vec{v}_{in} + \frac{A}{A+1}\,\vec{V}_{in}$$

$\vec{V}_{in}$ being zero (nuclei at rest in the laboratory system), this translates to:

$$\vec{v}_{CM} = \frac{1}{A+1}\,\vec{v}_{in}$$

So, going back to the laboratory system, it can be shown that the final kinetic energy of the neutron depends on the deflection angle θ and the nucleus mass A.

$$\frac{E_{out}}{E_{in}} = \frac{A^2 + 1 + 2A.\cos\theta}{(A+1)^2} \leq 1$$

There is a correlation between the outgoing energy and the deflection angle. The deflection angle is stochastic (random).

- $\cos\theta$ has a value between -1 and 1
- For $\theta = 0$, the neutron goes straight ahead, therefore there is no energy loss
- For $\theta = 180$, the neutron bounces backward, losing the maximum amount of energy.

$$\alpha E_{in} \leq E_{out} \leq E_{in}$$

Where $\alpha = \left(\frac{A-1}{A+1}\right)^2$

This parameter α is the minimum ratio between the final energy and the initial energy of the neutron, obtained when θ is equal to 0 or 180 degree. We can note that this value decreases as the mass of the target nucleus decreases, which shows that these nuclei are better at slowing down neutrons.

The maximum outgoing energy αE_{in} is zero for hydrogen ($A = 1$) and it increases with the mass of the target nucleus. Consequently:

- Light element are good moderator,
- Heavy elements don’t slow neutrons
- In hydrogen, the neutron can lose all its energy in a single collision (of course, on average, it will lose less)

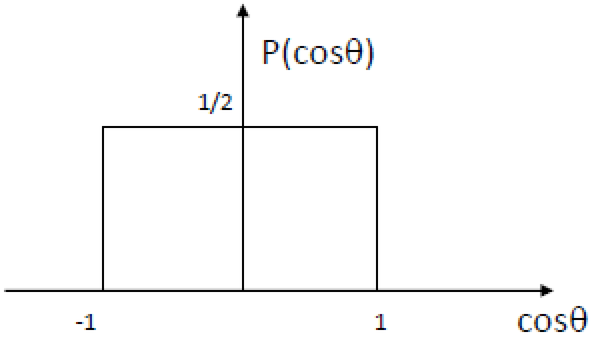
For isotropic scattering in the center of mass system, we have:

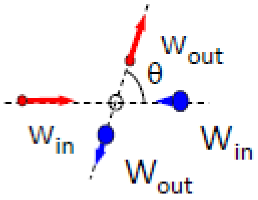
$$P(\theta,\phi)d\theta d\phi = \frac{\sin\theta\, d\theta d\phi}{4\pi} = \frac{d^2\Omega}{4\pi}$$

By writing $\mu = \cos\theta$ and thus $d\mu = \sin\theta . d\theta$,

$$P(\mu)d\mu = \frac{1}{2}d\mu$$

The distribution is uniform in $\cos\theta$ (deflection angle in the center of mass system):





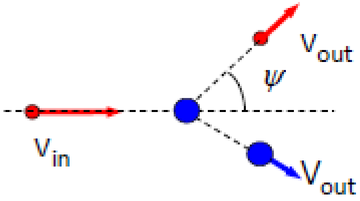
In the lab system, on the other hand, the distribution is no longer isotropic in $\cos\psi$ (deflection angle in the lab system), especially for light nuclei:

$$\cos\psi = \frac{1 + A.\cos\theta}{\sqrt{A^2 + 1 + 2A.\cos\theta}}$$

Therefore, the average value of $\cos\psi$ is:

$$< \cos\psi > = \frac{2}{3\mathrm{A}} \overset{\text{def}}{=} \bar{\mu}_0$$

This shows that the scattering is forward peeked in the lab system:



We can recall several things. The neutrons lose a fraction (α) of their energy at each collision and the slowing down region encompass more than six orders of magnitude (from MeV to eV). We now define a new variable, the lethargy u.

$$u = \ln\frac{E_0}{E}$$

$$du = -\frac{dE}{E}$$

Here, E_0 represents any reference energy. Considering it equaled to 20 MeV will give us values of the lethargy which are always positive.

When the energy goes down, the lethargy goes up. So, during the slowing down process, lethargy is an increasing function.

We can compute the average lethargy gain per collision according to:

$$w \stackrel{\text{def}}{=} \Delta u = -\ln \frac{A^2 + 1 + 2A.\cos\theta}{(A+1)^2}$$

Using $\alpha = \left(\frac{A-1}{A+1}\right)^2$:

$$w = -\ln\left\{\frac{1}{2}[1 + \alpha + (1-\alpha).\cos\theta]\right\}$$

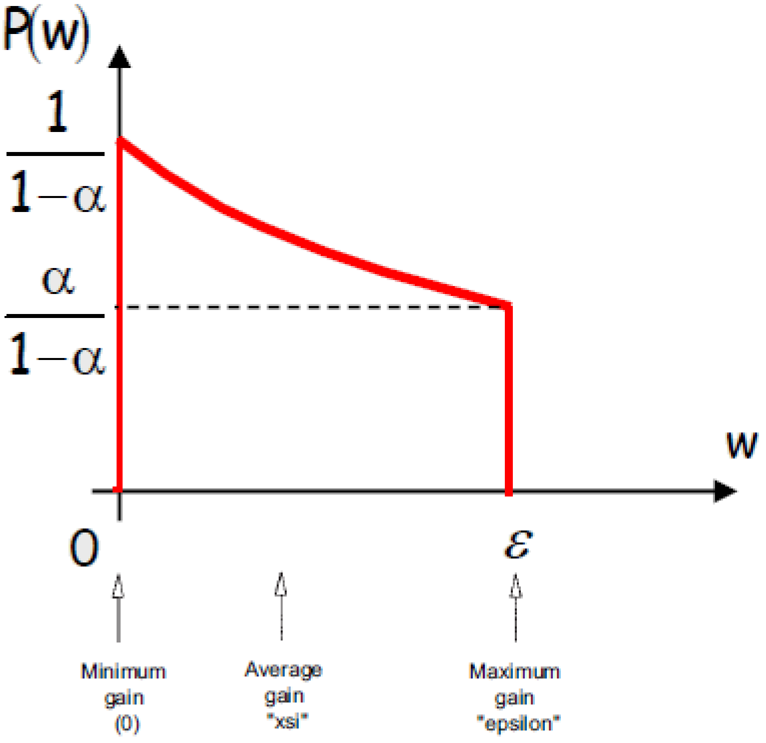
The maximal gain ε is obtained when the loss in energy for the neutron is maximum, therefore for $\theta = -180°$, and thus:

$$\varepsilon \stackrel{\text{def}}{=} w_{max} = -\ln\alpha$$

It is interesting to note that the probability distribution for the lethargy gain due to isotropic scattering is not uniform:

$$P(w)dw = P(E)dE$$

$$P(w) = \frac{\mathrm{e}^{-\mathrm{w}}}{1-\alpha} \qquad 0 \leq w \leq \varepsilon$$



This finally brings us to where we wanted to go. To slow down neutrons in a thermal reactor, one needs a good moderator. How is this moderator chosen?

We call slowing-down power (ξ) the average lethargy gain per collision:

$$\xi \overset{\text{def}}{=} < w > = \int_{\alpha E_{in}}^{E_{in}} dE_{out} w P(E_{out})$$

$$\xi = 1 - \frac{\alpha \varepsilon}{1 - \alpha}$$

For hydrogen, $\xi = 1$. We can compute the average number of collisions necessary to slow down a neutron for say $2\ MeV$ to $1\ eV$:

$$\Delta u = -\ln \frac{A^2 + 1 + 2A.\cos\theta}{(A+1)^2}$$

And:

$$\frac{E_{out}}{E_{in}} = \frac{A^2 + 1 + 2A.\cos\theta}{(A+1)^2}$$

So,

$$\Delta u = -\ln \frac{E_{out}}{E_{in}} = -\ln \frac{1}{2 * 10^6} = \ln(2 * 10^6) = 14.5$$

Consequently, the number of collision needed on average in this particular case is:

$$Nb = \frac{14.5}{\xi} = 14.5$$

In conclusion, we can see that a good moderator must be:

- Light
  - A low mass of the nucleus (low A) implies that the α defined earlier will be low, hence a high ξ.
- Diffusive
  - That means a high σ_s. The more scattering collisions we have, the faster the neutron will be thermalized.
- Non absorbant
  - This means a low σ_a, so that $\frac{\sigma_a}{\sigma_s} \ll 1$. We want to keep the neutrons for fission, so we don't want them to be absorbed "on the (energy) way"
- Dense
  - We want a high Σ_s ($= N\sigma_s$), to increase the probability of collision. The more targets we have, the higher the chances are for the neutrons to collide.

So, the value to watch for when deciding of our moderator are the slowing-down power ξ, the moderating power $\xi\Sigma_s$, and the ratio with the absorption $\frac{\xi\Sigma_s}{\Sigma_a}$.

The main candidates for moderation are water, heavy water, graphite and beryllium. The helium could have been a good candidate (light, low absorption), but because it exists only as a gas, it's not dense enough to be used as moderator. Uranium is indicated as a reference there.

Material	ξ	Nb $(2MeV \to 1eV)$	$\xi\Sigma_s$	$\frac{\xi\Sigma_s}{\Sigma_a}$
H_2O	0.920	16	1.35	71
D_2O	0.509	29	0.176	5670
Be	0.207	70	0.162	172
$C\ (graphite)$	0.158	91	0.06	192
U	0.008	1730	0.003	0.0092

What do we see in this table? Well, first, in terms of slowing down the neutrons, we can see that water is the best, followed by heavy water. Beryllium and graphite are still acceptable. What matters most is the last column. It shows that the absorption in water is quite high. We can moreover see that heavy water is very good at not absorbing neutrons. So, if we translate all this information into a more readable table, we obtain:

Material	State	Slowing	Capture	Cost	Natural Uranium
Water (H_2O)	Liquid	Excellent	Mediocre	Nada	Impossible
Heavy water (D_2O)	Liquid	Excellent	Excellent	High	Possible
Glucine (BeO)	Solid	Average	Good	Average	Possible
Graphite (C)	Solid	Average	Good	Average	Possible

Careful here, by "mediocre" in the capture column, I don't mean that the capture is mediocre, but, au contraire, that it is very high, thus a quite bad thing for a moderator. It absorbs a lot of neutrons, and we do not want that.
If both columns 3 and 4 are favorable, a natural uranium (not enriched) reactor is possible. We can see that this is the case for heavy water, glucine and graphite.

However, you might already know that nowadays, water is mostly used as a moderator in nuclear reactors. Few designs use heavy water (looking at you, Canada), and fewer still use graphite. I've never heard of any reactors running with Beryllium as a moderator.

So, why is it that water is mostly used? Well, if we forget this annoying high absorption cross-section of the hydrogen, we have several key advantages.

1. The cost. Really, this is an industry. Using water in your reactor only requires of you that you perform a purity check.
2. The thermodynamic properties. We know everything about water. And it can conveniently act as the coolant!
3. Awesome moderating power (A factor of more than 9 with the heavy water). This parameter turns out to be the best measure of the material's ability to slow down

neutrons. Thanks to this moderating power, we can have very compact reactor with water.

That said, the use of water comes with one big disadvantage: we need to use an enriched fuel.

Well, this ends the sixth lecture. If you have any question, please let me know directly or post a thread in the dedicated subreddit. Do not forget, and I can't stress this enough: if you have a question, then someone else in the class is wondering the same thing, or should be. Therefore, asking it will help you and others.

I highly recommend that you actually do the math. I did not show every single step, and it would be very beneficial for you to take over the equations and make them yours, as it helps you clear things up. It also requires some effort, but that's the price of knowledge, isn't it?

The next lecture is tightly intertwined with this one, mathematically speaking.

Once again, there is another thing that I should repeat. If you do not understand something, do not feel like it's your fault, and do not give up. It merely means that my explanations were not good enough. I will gladly upgrade the class by taking into account your suggestions and remarks.